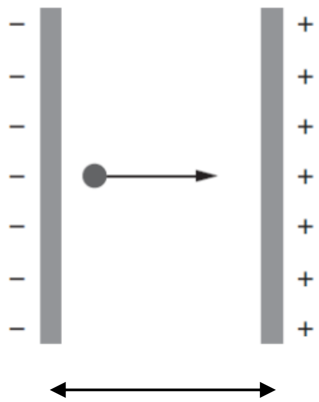
Problem of the week

Motion in E&M fields

(a) Two parallel plates are oppositely charged. The potential difference between them is 240 V and their separation 8.0 cm.



A proton is launched from the negative plate with speed u. The proton just reaches the positive plate and instantaneously stops before turning back.

Determine, for the proton,

(i) the acceleration,
(ii) the initial speed u,
(iii) the time to reach the positive plate.

(b) The distance between the plates in (a) is doubled but the potential difference stays the same. State and explain how the answers to (i), (ii) and (iii) in (a) change, if at all.

(c) The proton in (a) is replaced by an alpha particle. The alpha particle just reaches the positive plate. The separation of the plates is 8.0 cm and the potential difference is 240 V. State and explain how the answers to (i), (ii) and (iii) in (a) change, if at all.

(d) A proton enters the region between two parallel oppositely charged plates mid-way between the plates.

The following data are available:

Potential difference between plates	60 V
Length of plates	28 cm
Separation of plates	12 cm
Initial proton speed	2.0×10^{5} m s^{-1}

(i) Calculate the time it takes the proton to cover a horizontal distance equal to the length of the plates.

(ii) For the time in (i) determine the vertical distance covered by the proton.

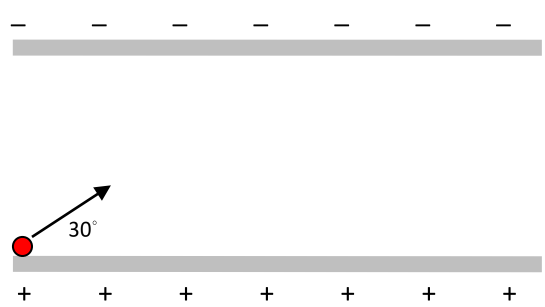
(iii) Hence determine whether the proton will hit one of the plates.

(iv) Draw the path of the proton.

(e) Two parallel plates are oppositely charged. The potential difference between them is V and are separated by a distance d. A particle with a negative charge of magnitude q is moved from the negative to the positive plate.

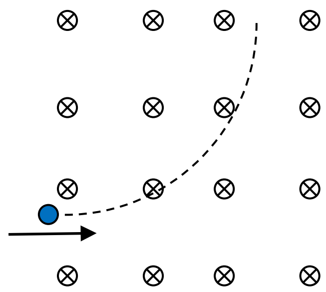
(i) Show that the work done is $W = qV$.

(ii) The same particle is now launched as shown, at an angle 30° to the horizontal. Gravity is ignored. The kinetic energy of the particle at launch is equal to W, the work in (i).



Determine the maximum distance of the particle from the lower plate.

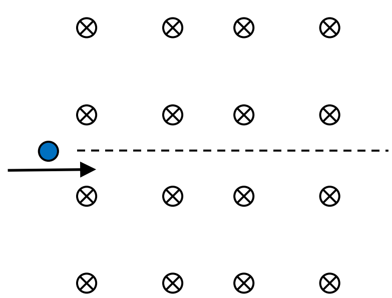
(f) A proton enters a region of uniform magnetic field of 0.80 T with speed 3.6×10^{6} m s^{-1}.



The proton exits the region of magnetic field after completing a quarter of a circle.

(i) State and explain the speed of the proton as it exits the region of magnetic field.
(ii) Determine the time the proton stayed in the region of magnetic field.
(iii) Calculate the magnitude of the average magnetic force experienced by the proton.

(g) An electric field is now established in the region of magnetic field as shown so that the proton moves along a straight-line path.



(i) Draw, on the diagram above, field lines for this electric field.
(ii) Determine the magnitude of the electric field strength.
(iii) The proton is replaced by an electron of the same velocity. State and explain the path of the electron.

Answers

(a)

(i) $a = \frac{F}{m} = \frac{eE}{m} = \frac{e}{m}\frac{V}{d} = \frac{1.6\times10^{-19}\times240}{1.67\times10^{-27}\times8.0\times10^{-2}} = 2.874\times10^{11} \approx 2.9\times10^{11}\text{ m s}^{-2}$.

(ii) From $v^2 = u^2 - 2as$ we get

$0 = u^2 - 2\frac{e}{m}\frac{V}{d}d \Rightarrow u = \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2\times1.6\times10^{-19}\times240}{1.67\times10^{-27}}} = 2.144\times10^{5} \approx 2.1\times10^{5}\text{ m s}^{-1}$.

(iii) $v = u - at = u - \frac{eV}{md}t \Rightarrow t = \frac{mdu}{eV} = \frac{1.67\times10^{-27}\times8.0\times10^{-2}\times2.144\times10^{5}}{1.6\times10^{-19}\times240} = 7.461\times10^{-7} \approx 7.5\times10^{-7}\text{ s}$

(b) From $a = \frac{e}{m}\frac{V}{d}$, the acceleration halves. From $u = \sqrt{\frac{2eV}{m}}$, the speed is unchanged. From $t = \frac{mdu}{eV}$, the time doubles.

(c) From $a_\alpha = \frac{2e}{4m}\frac{V}{d} = \frac{1}{2}a_p$. From $u_\alpha = \sqrt{\frac{2\times2eV}{4m}} = \frac{1}{\sqrt{2}}\sqrt{\frac{2eV}{m}} = \frac{u_p}{\sqrt{2}}$. From

$t_\alpha = \frac{4md\frac{u_p}{\sqrt{2}}}{2eV} = \frac{2}{\sqrt{2}}t_p = t_p\sqrt{2}$.

(d)

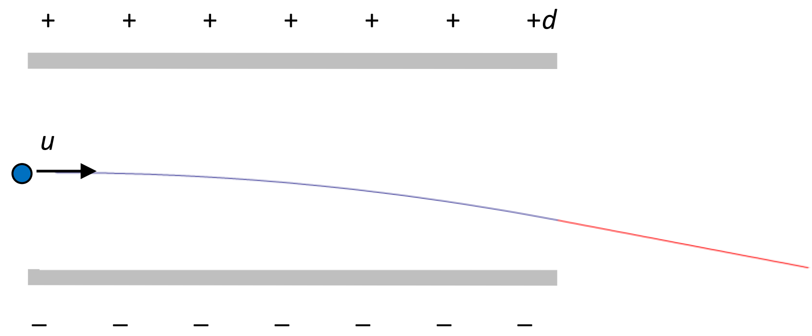
(i) $t = \frac{L}{u} = \frac{0.28}{2.0\times10^{5}} = 1.4\times10^{-6}\text{ s}$.

(ii) $a = \frac{eV}{md} = \frac{1.6\times10^{-19}\times60}{1.67\times10^{-27}\times12\times10^{-2}} = 4.791\times10^{10} \approx 4.8\times10^{10}\text{ m s}^{-2}$.

$y = \frac{1}{2}at^2 = \frac{1}{2}\times4.791\times10^{10}\times(1.4\times10^{-6})^2 = 4.920\times10^{-2}\text{ m} \approx 4.9\text{ cm}$.

(iii) No, the proton moved down 4.9 cm and the distance between the lower plate and the middle of the plates is 6.0 cm.

(iv) Parabolic path within the plates, straight line path outside the plates.



(e)

(i) $W = Fd = qEd = q\frac{V}{d}d = qV$.

(ii) Let the initial speed be u. Then, at the maximum distance from the lower plate $v_y = 0 = u\sin\theta - at \Rightarrow t = \frac{u\sin\theta}{a}$. But $a = \frac{qV}{md}$ and so $t = \frac{u\sin\theta}{\frac{qV}{md}} = \frac{mdu\sin\theta}{qV} = \frac{1}{2}\frac{mdu}{qV}$ since $\theta = 30^\circ$. The maximum distance is then $y_{\max} = ut\sin 30^\circ - \frac{1}{2}at^2 = \frac{u}{2}\times\frac{1}{2}\frac{mdu}{qV} - \frac{1}{2}\frac{qV}{md}\times(\frac{1}{2}\frac{mdu}{qV})^2 = \frac{mu^2}{4qV}d - \frac{mu^2}{8qV}d = \frac{mu^2}{8qV}d$. In other words, $y_{\max} = \frac{2E_K}{8W}d = \frac{d}{4}$, since $E_K = W$.

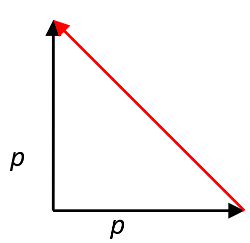
(f)

(i) The magnetic force is at right angles to the velocity and so does zero work. Hence the kinetic energy and speed do not change.

(ii) The radius of the path is found from: $evB = \frac{mv^2}{R} \Rightarrow R = \frac{mv}{eB}$. Hence the time is

$$T = \frac{1}{4}\frac{2\pi R}{v} = \frac{1}{4}\frac{2\pi}{v}\frac{mv}{eB} = \frac{\pi m}{2eB} = \frac{\pi\times 1.67\times 10^{-27}}{2\times 1.6\times 10^{-19}\times 0.80} = 2.0\times 10^{-8}\ \text{s}.$$

(iii) The magnitude of the momentum change of the proton is shown by the red arrow.

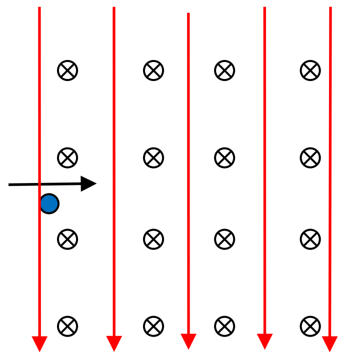


It is $\Delta p = p\sqrt{2} = 1.67\times 10^{-27}\times 3.6\times 10^{6}\times\sqrt{2} = 8.502\times 10^{-21}\ \text{N s}$. Hence the average force is

$$\frac{\Delta p}{\Delta t} = \frac{8.502\times 10^{-21}}{2.0\times 10^{-8}} = 4.3\times 10^{-13}\ \text{N}.$$

(g)

(i)



(ii) $eE = evB \Rightarrow E = vB = 3.6\times 10^6 \times 0.80 = 2.9\times 10^6\ \text{N C}^{-1}$.

(iii) The electron will follow the same straight-line path. The electric and magnetic forces on the electron will be the reverse of those on the proton but they will be equal in magnitude and so cancel out.